Variational and Extremum Principles in Macroscopic Systems

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Book reviews

Theory of Neural Information Processing Systems
A C C Coolen, R Kühn and P Sollich
2005 Oxford University Press
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It is difficult not to be amazed by the ability of the human brain to process, to structure and to memorize information. Even by the toughest standards the behaviour of this network of about 10^{11} neurons qualifies as complex, and both the scientific community and the public take great interest in the growing field of neuroscience.

The scientific endeavour to learn more about the function of the brain as an information processing system is here a truly interdisciplinary one, with important contributions from biology, computer science, physics, engineering and mathematics as the authors quite rightly point out in the introduction of their book. The role of the theoretical disciplines here is to provide mathematical models of information processing systems and the tools to study them. These models and tools are at the centre of the material covered in the book by Coolen, Kühn and Sollich.

The book is divided into five parts, providing basic introductory material on neural network models as well as the details of advanced techniques to study them. A mathematical appendix complements the main text. The range of topics is extremely broad, still the presentation is concise and the book well arranged. To stress the breadth of the book let me just mention a few keywords here: the material ranges from the basics of perceptrons and recurrent network architectures to more advanced aspects such as Bayesian learning and support vector machines; Shannon's theory of information and the definition of entropy are discussed, and a chapter on Amari's information geometry is not missing either. Finally the statistical mechanics chapters cover Gardner theory and the replica analysis of the Hopfield model, not without being preceded by a brief introduction of the basic concepts of equilibrium statistical physics. The book also contains a part on effective theories of the macroscopic dynamics of neural networks. Many dynamical aspects of neural networks are usually hard to find in the existing textbook literature, so that this discussion will be very much appreciated.

The book is of an exceptionally high quality in all aspects. In my view, the style of presentation and the inclusion of recent aspects of the topic alone make the book a welcomed addition to the existing literature. It is well structured and the material covered was chosen with care. While focusing on quantitative aspects of the subject, the authors adopt a comprehensive style of presentation, being precise, but not pedantic. The student who is not familiar with the field might find the breadth of the book overwhelming at first, but will soon appreciate its pedagogical value. All mathematical derivations are performed and explained step by step for the student to follow, and they are illustrated by many concrete examples and results from computer simulations in well-presented and clear figures. If a student wants to get his hands on the mathematical tools of neural networks theory then this book is a good place to learn from. A set of instructive and valuable exercises complements each chapter (hints are given, but maybe it would have been nice to provide additional brief sample solutions in an appendix). I very much enjoyed the outlook sections at the end of each of the five parts, putting the material covered into its historical context and providing further references.

In summary, students of a quantitative discipline will find in this book a clear and self-contained introduction to the subject, lecturers might use it to design postgraduate courses, and finally it will provide a valuable reference for researchers working in the area. This book can be expected to be an asset for all types of readers, even if they already own a book on neural networks. Anyone with a serious interest in the theoretical aspects of the field would be making a mistake not to have a copy on their shelves.

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Introduction to Computational Plasticity
F Dunne and N Petrinic
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The use of computational modelling in all areas of science and engineering has in recent years escalated to the point where it underpins much of current research. However, the distinction must be made between computer systems in which no knowledge of the underlying computer technology or computational theory is required and those areas of research where the mastery of computational techniques is of great value, almost essential, for final year undergraduates or masters students planning to pursue a career in research. Such a field of research in the latter category is continuum mechanics, and in particular non-linear material behaviour, which is the core topic of this book. The focus of the book on computational plasticity embodies techniques of relevance not only to academic researchers, but also of interest to industrialists engaged in the production of components using bulk or sheet forming processes. Of particular interest is the guidance on how to create modules for use with the commercial system Abaqus for specific types of material behaviour.

The book is in two parts, the first of which contains six chapters, starting with microplasticity, but predominantly on continuum plasticity. The first chapter on microplasticty gives a brief description of the grain structure of metals and the existence of slip systems within the grains. This provides an introduction to the concept of incompressibility during plastic deformation, the nature of plastic yield and the importance of the critically resolved shear stress on the slip planes (Schmid's law). Some knowledge of the notation commonly used to describe slip systems is assumed, which will be familiar to students of metallurgy, but anyone with a more general engineering background may need to undertake additional reading to understand the various descriptions. Any lack of knowledge in this area however, is of no disadvantage as it serves only as an introduction and the book moves on quickly to continuum plasticity. Chapter two introduces one of several yield criteria, that normally attributed to von Mises (though historians of mechanics might argue over who was first to develop the theory of yielding associated with strain energy density), and its two or three-dimensional representation as a yield surface. The expansion of the yield surface during plastic deformation, its translation due to kinematic hardening and the Bauschinger effect in reversed loading are described with a direct link to the material stress-strain curve. The assumption, that the increment of strain is normal to the yield surface, the normality principle, is introduced. Uniaxial loading of an elastic-plastic material is used as an example in which to develop expressions to describe increments in stress and strain. The full presentation of numerous expressions, tensors and matrices with a clear explanation of their development, is a recurring, and commendable, feature of the book, which provides an invaluable introduction for those new to the subject. The chapter moves on from time-independent behaviour to introduce viscoplasticity and creep. Chapter three takes the theories of deformation another stage further to consider the problems associated with large deformation in which an important concept is the separation of the phenomenon into material stretch and rotation. The latter is crucial to allow correct measures of strain and stress to be developed in which the effects of rigid body rotation do not contribute to these variables. Hence, the introduction of 'objective' measures for stress and strain. These are described with reference to deformation gradients, which are clearly explained; however, the introduction of displacement gradients passes with little comment, although velocity gradients appear later in the chapter. The interpretation of different strain measures, e.g. Green–Lagrange and Almansi, is covered briefly, followed by a description of the spin tensor and its use in developing the objective Jaumann rate of stress. It is tempting here to suggest that a more complete description should be given together with other measures of strain and stress, of which there are several, but there would be a danger of changing the book from an 'introduction' to a more comprehensive text, and examples of such exist already. Chapter four begins the process of developing the plasticity theories into a form suitable for inclusion in the finite-element method. The starting point is Hamilton's principle for equilibrium of a dynamic system. A very brief introduction to the finite-element method is then given, followed by the finite-element equilibrium equations and a description of how they are incorporated into Hamilton's principle. A useful clarification is provided by comparing tensor notation and the form normally used in finite-element expressions, i.e. Voigt notation. The chapter concludes with a brief overview of implicit integration methods, i.e. tangent stiffness, initial tangent stiffness and Newton–Raphson. Chapter five deals with the more specialized topic of implicit and explicit integration of von Mises plasticity. One of the techniques described is the radial-return method which ensures that the stresses at the end of an increment of deformation always lie on the expanded yield surface. Although this method guarantees a solution it may not always be the most accurate for large deformation, this is one area where reference to alternative methods would have been a helpful addition. Chapter six continues with further detail of how the plasticity models may be incorporated into finite-element codes, with particular reference to the Abaqus package and the use of user-defined subroutines, introduced via a 'UMAT' subroutine. This completes part I of the book.

Part II focuses on plasticity models, each chapter dealing with a particular process or material model. For example, chapter seven deals with superplasticity, chapter eight with porous plasticity, chapter nine with creep and chapter ten with cyclic plasticity, creep and TMF. Examples of deep drawing, forming of titanium metal-matrix composites and creep damage are provided, together with further guidelines on the use of Abaqus to model these processes.

Overall, the book is organised in a very logical and readable form. The use of simple one-dimensional examples, with full descriptions of tensors and vectors throughout the book, is particularly useful. It provides a good introduction to the topic, covering much of the theory and with applications to give a good grounding that can be taken further with more comprehensive advanced texts. An excellent starting point for anyone involved in research in computational plasticity.

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Variational and Extremum Principles in Macroscopic Systems
Stanislaw Sieniutycz and Henrik Farkas (eds)
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Given a dissipative system, is there a variational theory describing it? A significant part of the book is devoted to this question, though other aspects of variational principles are covered as well. The first half of the book covers theory, while the second presents a wide range of applications of variational principles.

Variational principles are more commonly associated with conservative systems. An example of a classic dissipative system in variational guise might thus be in order. Consider the motion of a particle in a potential field under the influence of linear friction,

$$m\ddot{x} = -DV(x,t) - \lambda m\dot{x}. \tag{1}$$

Equation (1) can easily be seen to be equivalent to the Euler-Lagrange equation for Bateman's Lagrangian

$$L(x,\dot{x},t) := \left(\frac{m}{2}\dot{x}^2 - V(x,t)\right)e^{\lambda t}.$$

For a scientist studying a dissipative system, finding a variational principle is a dream, as powerful tools from the Calculus of Variations and numerical analysis become available. In practise, the quest for variational formulations for dissipative systems seems more like a nightmare: our understanding is not nearly as satisfactory and coherent as we would like it to be. The book makes a valuable contribution by presenting various recent approaches side by side, giving equal voice to each of them. Maugin and Kalpakides establish a variational form for classical thermoelasticity of anelastic conductors. The key ingredient here is the inclusion of thermacy to the list of fields; thermacy is the time primitive of the temperature. Anthony sketches the Lagrangian formalism for irreversible thermodynamics. Here, in analogy to quantum mechanics, complex-valued field variables are introduced, such as the field of thermal excitation, from which the absolute temperature can be derived as the squared modulus. Anthony focuses in particular on the second law of thermodynamics and the principle of least entropy production. Different ways of creating Lagrangians for dissipative systems are due to Sieniutycz and Berry (based on symmetry principles and conservation laws), and Gambár and Márkus. The book is a good reference for these results, and contains papers of most of these authors. For example, Gambár's and Márkus' contributions to the book include a discussion of action principles for parabolic equations and the quantization procedure for heat conduction. Wagner discusses the derivation of Lagrangians for hydrodynamic systems with linear damping, exploiting an analogy between hydrodynamics and the hydrodynamic picture of quantum theory.

Variational principles also appear in very different situations: for example, the speed of a travelling wave for the reaction-diffusion equation can under suitable assumptions be characterized in a variational manner. Benguria and Depassier give a very readable review, including motivations and proofs, of some results in this direction.

It is, within the space of a review, not possible to do justice to all contributions. Suffice it to say that a wide range of topics is covered in the part discussing theory, including relativistic aspects.

Both theory and applications are given about the same weight and space. The second half of the book is devoted to applications and starts with a short section of statistical physics and thermodynamics. Alternatives are presented to

Jaynes' approach of statistical mechanics (the maximum entropy method, where the probability density function is derived from the entropy through extremization). Plastino, Plastino and Casas show that thermostatistics can equally be derived via a constrained extremization of Fisher's information measure. A clear overview of a different line of thought is given by de Almeida. The entropy is derived here from the probability density as a consequence of the second law of thermodynamics. Based on data relating the Reynolds number and entropy production, Paulus and Gaggioli hypothesize that, for a specified mass flow rate, a stable fluid flow corresponds to a maximal rate of entropy production, while a stable flows corresponds for a specified pressure drop to a minimal entropy production rate.

Presumably, many readers will not be familiar with variational principles in ecology and economics. Jørgensen's paper may serve as an introduction to variational principles in ecology. The fundamental concept discussed here is ecoexergy, which measures the distance from thermodynamic equilibrium. It is argued that maximization of ecoexergy occurs in ecosystems; however, different maximization criteria such as emergy are discussed as well.

These sketches should give a flavour of the applications discussed; the range also includes fluid mechanics (e.g., drag in linear hydrodynamics), continuum mechanics (e.g., local and global buckling of elastic plates with composite structure), and transport phenomena. The mathematical methods employed here are often classical tools from engineering mathematics. The emphasis is on the derivation of variational descriptions for the applications under consideration.

The book sets out to serve as a reference for researchers in applied mathematics, macroscopic physics and chemistry as well as a source for graduate students in related fields. I feel that both goals have been accomplished. Occasional imprecision, such as the lack of a clear distinction between minimal and stationary principles in some contributions, do not destroy the overall impression that this is a thoughtfully selected and carefully edited compilation. Many of the mathematical concepts presented in this collection of papers (Hamiltonian and Lagrangian formalism, Noether's theorem) are available in a concise form in standard textbooks. The valuable contribution of the book resides in the presentation of recent developments, particularly of variational theories for dissipative systems, as well as the wealth of applications. This should make the study of the book under review a worthwhile endeavour for students. The breadth of topics and approaches under discussion recommends the book to scientists and applied mathematicians alike.

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